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Sampling Landings of Halibut for Age Composition

by

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FOREWORD

In 1972, IPHC began compiling historical statistics and cataloguing analytical procedures that had not been published. This undertaking included the reexamination of earlier interpretations of data pertinent to the management of the fishery. As these tasks are completed, they will appear in IPHC's Scientific and Technical Report series. This paper is the fourth of these projects, the others were IPHC Scientific Report No. 54 and No. 56 and Technical Report No. 10.

ABSTRACT

The International Pacific Halibut Commission (IPHC) has sampled landings of halibut since the mid-1930's. Statistical divisions along the coast were defined by IPHC to facilitate the recording of landing statistics. Landing data as well as length and age data are analyzed for groups of these statistical divisions. Initially, only landings from certain "indicator" grounds were sampled. The present age and length sampling is based on a stratified design extending over the entire range of the fishery. To maintain continuity with early series of data, IPHC has accepted a month as a unit of time, and the historical groupings of statistical divisions are accepted as the areal strata.

A systematic sampling scheme to provide random selection of landings is discussed. Double sampling for length and age data, as it was applied previously and as it is being applied in an improved design, is discussed. Sample sizes are determined for different levels of precision in estimation of relative age frequency distribution. A procedure for obtaining a weighted average age distribution for regulatory areas is given.

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INTRODUCTION

The North Pacific halibut fishery has passed through the initial exploratory phase common to all fisheries and is a fully developed, intensive fishery whose geographical distribution approximates the distribution of the adult halibut *(Hippoglossus stenolepis)* stock. The International Pacific Halibut Commission (IPHC) has managed the fishery for more than 50 years and uses length and age data from commercial landings to interpret changes in halibut populations. The limitations of relying on a commercial fishery to sample a population randomly are well known. The gear does not take all sizes of halibut with equal efficiency (Myhre, 1969); hence, samples drawn from the fishery are biased. Because of the great geographical range of halibut — approximately 2,500 miles, IPHC considers the setline fleets of Canada and the United States the primary sampling device, despite the selectivity of the gear, and samples the landings of halibut at several ports.

Samples of commercial landings began in the mid-1930's; double sampling techniques (Cochran, 1963) were employed. Data from some grounds span many years and have been obtained and analyzed in a consistent manner, whereas data from other grounds have been collected only in recent years. The data were used to ascertain age frequencies from which growth and mortality were determined. These rates were applied directly in mathematical models of the resource. IPHC decided that changes in length and age frequencies of the halibut on certain ''indicator'' grounds were representative of changes in the stocks as a whole, and sampling was limited to these grounds. This program prevailed until the late 1940's when it was expanded to include other grounds. In recent years, several weighting procedures have been used to estimate age frequencies of the landings in each regulatory area. These weighting procedures raised questions regarding areal stratification, size of sample, and methods of combining the data. The need for continuity in the age and length data series predicated use of time and areal stratifications which were logical extensions of those used earlier. The earlier strategy of double sampling for age data, and the method of combining these data to obtain a weighted age frequency by area, is compared with a new strategy of double sampling which reduces variability of the relative age frequency.

STATISTICAL DIVISIONS AND REGULATORY AREAS

Pacific halibut are fished commercially from California to the Bering Sea. In 1931, IPHC defined statistical divisions along the coast to facilitate analysis of data from the fishery (Thompson, Dunlop, and Bell, 1931). Each division was 60 miles wide and was defined by lines running perpendicular to the coast (Figure 1). These statistical divisions do not cover equal areas of fishing grounds. Regulatory areas were defined in terms of statistical divisions, and landings were analyzed according to groupings of statistical divisions (Bell, Dunlop, and Freeman, 1952). These groupings are as follows:

Regulatory Area 2	Statistical Divisions
South of Willapa Harbor, Washington	000-030
Willapa Harbor to Cape Scott (north end of Vancouver Island)	040-080
Cape Scott to Dixon Entrance	090-130
Dixon Entrance to Cape Spencer	140-183

Regulatory Area 3	Statistical Divisions
Cape Spencer to Cape St. Elias	185-230
Cape St. Elias to Trinity Islands	240-280
Trinity Islands and West	290+

The grounds from Cape Scott to Cape Spencer (090-183) include protected inside waters among the channels and bays, as well as open ocean or outside waters. Because the catches of halibut taken in outside waters have had a higher proportion of older fish than those taken in inside waters (IPHC, 1961; 1973), the region between Cape Scott and Cape Spencer was divided into "inside" and "outside" sections.

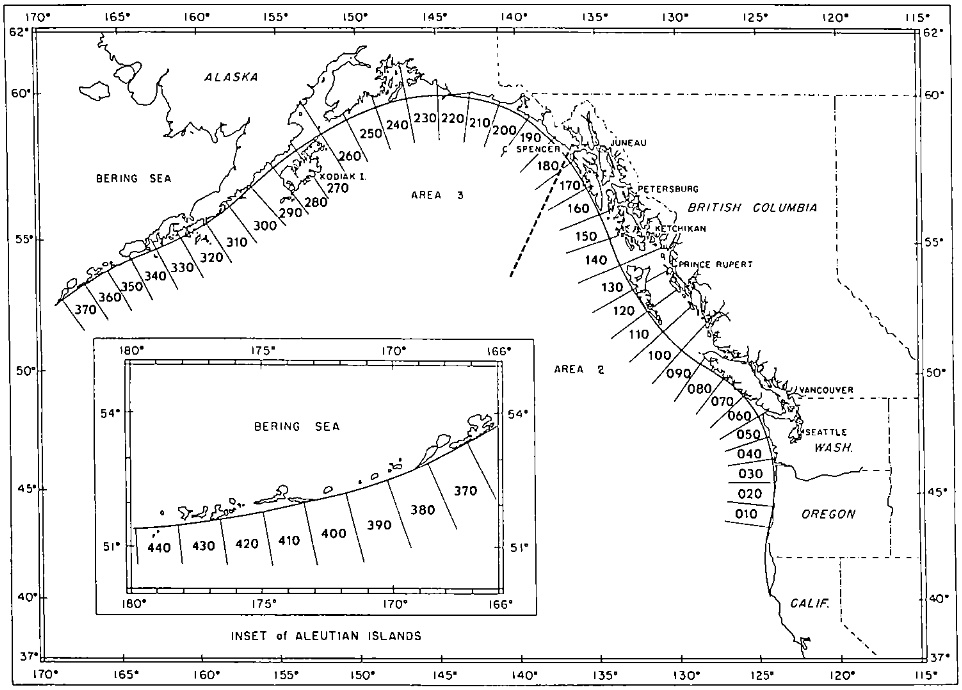

Figure 1. Statistical divisions and regulatory areas used for the Pacific halibut fishery.

SAMPLING TECHNIQUES

Halibut are eviscerated when caught and packed in the hold of the catching vessel; their storage location determines the trim of the vessel. Although daily fishing information is available, the location of halibut in the hold of the vessel cannot be used in a sampling design because each day's catch is not identified, and storage procedures differ from vessel to vessel. To unload the catch, two or three crewmen fill a cargo sling with halibut; the sling is lifted to a heading table, about 10 feet square, on the unloading dock. A sling holds approximately 1,000 pounds of halibut, and a single vessel landing may exceed 100,000 pounds. The heads of the fish are removed, either with knives or a mechanical header (guillotine). To collect otoliths for estimating length and age, the sampling crew must intercept the fish before they are beheaded.

In the early years, a four-man crew sampled landings at each sampling location. The age sample (from otoliths) was a subsample of the catch, as was the length sample (fish measurements); both were taken simultaneously. Studies in the 1950's showed that fish length could be estimated from otolith measurements, and only one person was needed to sample a landing (Southward and Hardman, 1973). With this technique, the age sample was a subsample of the length sample, both being obtained from otoliths. This change permitted proper double sampling and sampling at plants that could not accommodate a four-man crew. Both systems required a random sample, but enumerating the fish in the cargo sling and using random numbers to choose fish for measurement was not practical. Rather, a ''grab'' sample was taken; in this procedure, the person sampling the catch chose 6 to 10 fish from a sling-load and attempted to sample each sling. Often the fish chosen were those closest to the sampler, or those that fell in a certain position on the table. The potential for bias was great, the randomness was questionable, and IPHC could not be sure that the procedure was standardized from crew to crew. Currently, IPHC collects the left otolith from every fish in a randomly chosen sling. This method is discussed under the heading ''Within-Vessel Sampling — Grab Versus Sling Techniques''.

Time — Area Strata

From 1935 until 1949, IPHC used two indicator grounds: Goose Islands in Area 2 and Portlock-Albatross in Area 3. Trips were chosen for sampling primarily on the basis of their fishing location; only those trips which originated from one of these grounds were sampled. Landings from the indicator grounds were considered representative of the halibut stocks in the regulatory areas (Hardman and Southward, 1965). By the late 1940's, grounds in Hecate Strait had become more productive (IPHC, 1951). The trend toward progressively shorter seasons caused a shift in the distribution of fishing effort, which resulted in a change in the distribution of landings. The shortcomings of the two-indicator-ground design became apparent. Additional indicator grounds were established in Hecate Strait in 1949. Grounds in southeastern Alaska were added in 1958. Age data for these indicator grounds were combined and analyzed in monthly periods, as well as for the total season. In recent years, landings from other grounds have been sampled and these have been included in the combinations representing the various sections of the coast. Unfortunately, the longer series of age data represents a changing mixture of grounds.

As mentioned earlier, historical catch statistics have been tabulated by groupings of statistical divisions. These groupings were basic to the analysis of the catch data. To maintain continuity with earlier series of age data, a month was accepted by IPHC as a suitable stratum of time, and the groupings of statistical divisions were accepted as the areal stratum in the sampling design discussed here. The sampling universe considered in this study was the total commercial landings. Halibut as young as 2 or 3 years of age occupy the same grounds as older fish, but these young halibut are not caught by the setline fishery. Whether the commercial landings represent the halibut populations available for exploitation depends on the distribution of the fishing effort and the selectivity of the gear.

Sampling the Landings

Estimates of minimum variance rely on proportional allocation of intra-strata sample size. The allocation may be based on poundage landed within the areal strata, on intra-strata landing variances, or on the cost of obtaining the samples. If landings of individual vessels in the sample from each areal stratum were proportional to total landings from that stratum, the estimates would be self-weighting. Unfortunately, knowledge of the distribution of landings by areal strata, or reliable estimates of the intra-strata landing variances, are not available, nor can they be obtained prior to sampling because of the mobility of the vessels. The fishing captains respond quickly to changing market conditions among the ports. Therefore, the objective of past and present sampling programs has been to obtain a random sample of the landings in each major port.

IPHC personnel currently are assigned to sample commercial landings and are stationed in Seattle, Vancouver, Prince Rupert, Petersburg, Sitka, Kodiak, and Seward. Approximately 75% of the total catch is landed at these ports, and samples obtained from these ports are assumed to be representative of the total landings.

The occurrence of any vessel in port on a given day is assumed to be a random event, and a systematic sample of landings with a randomly selected starting point will result in a simple random sample at that port. Landings are stratified by poundage: more than 5,000 pounds, between 1,000 and 5,000 pounds, and less than 1,000 pounds. Choosing the vessel to be sampled is facilitated by maintaining a record of all landings within each poundage strata. Then, in order of unloading, every k^{th} vessel with a catch of 5,000 pounds or more, say every third vessel, is sampled. A large fleet of small vessels, generally salmon gillnetters, also fish (with setline gear) for halibut on an intermittent basis. Even though the net tonnage of the small vessels and their method of fishing differ from large setline vessels, they often fish the same grounds. Most of the small vessels usually land between 1,000 and 5,000 pounds per trip, and every tenth landing is sampled. No sample is taken of landings smaller than 1,000 pounds. The effect of omitting these landings (only 5% of the total in 1973) does not introduce a serious bias in the results. Comparison by statistical divisions of the percentages of total landings and every third landing is shown for 1973 landings in Prince Rupert (Figure 2).

Most of the landings in Prince Rupert are from statistical divisions 090-134 and 185-231, and the one-in-three sampling scheme adequately represented the total landings. The lack of close agreement in the other statistical divisions apparently is due to sampling problems caused by the small number of landings from these divisions. The decision to sample every third landing over 5,000 pounds and every tenth landing between 1,000 and 5,000 pounds was made to approximate the rate of sampling in previous years. Because of the difficulties in selecting representative samples when only a few data points are available, the representativeness of the one-in-three sampling must be examined periodically with data from all sampling ports to determine if the areal strata are included in the sample in their proper proportion.

The above rate of sampling will provide reliable variances of the relative age frequency. If IPHC finds that the variability is so large that the resulting estimates of mortality are not meaningful in their management program, the rate of sampling can be increased.

Skud (1972) has shown that the "mean weight of halibut increased as the hook-spacing increased" and that a trend toward greater distances between hooks is prevalent in the fleet. However, examination of the 1973 age distributions with respect to hook-spacing was inconclusive because the variability of these data was large and the average ages differed considerably among hook-spacings and grounds. Consequently, I have treated hook-spacing as a second order factor and did not consider it in the development of a new sampling design.

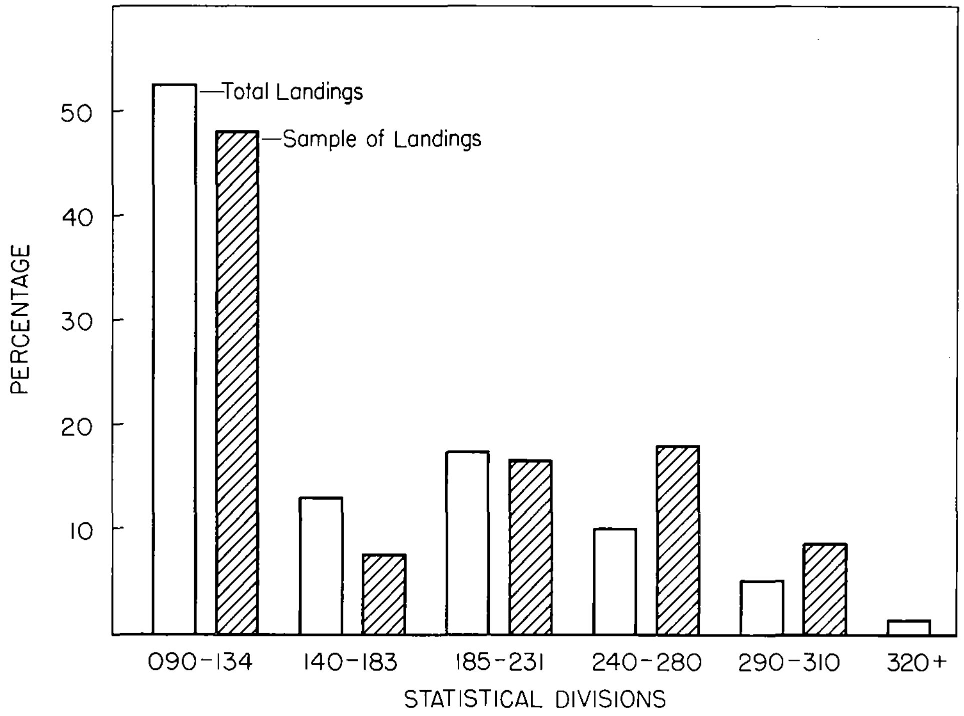

Figure 2. Percentage of total landings by statistical division and landings of every third vessel, Prince Rupert, 1973.

Within-Vessel Sampling — Grab Versus Sling Techniques

The grab technique, used for most of the sampling, did not utilize identifiable units of fish; rather, it required that the sampler randomly choose fish from a sling of halibut. The potential for personal biases was great. In 1971, as an alternative to the grab sample, Skud (unpublished) began testing the sling sample technique in which every fish in a randomly chosen sling is used in the sample. The sling technique utilizes the clustering effect and identifiable character of the sling.

Halibut are stored in the hold of the vessel in large bins; crewmen place halibut in several bins simultaneously. During the unloading of the catch, halibut are removed from the hold by means of a large cargo sling; two to four fishermen work in the hold of the vessel filling the sling. Halibut in any given sling are taken from several locations in the hold, thereby disrupting any fish size-location correlation which might exist. Also, any correlation between day caught and location in the hold is disrupted. Thus, in developing this sampling design, I considered the order in which halibut reach the heading table to be random with respect to the order in which they are caught. A systematic choice of sling loads within a vessel with a random choice of the first sling constitutes a simple random sample. Because slings can be chosen randomly and the selection of slings can be standardized among the sampling crews, a sling sample is preferable to a grab sample.

The representativeness of a sling sample with respect to a vessel's catch was examined using data from eight commercial trips. All fish in each trip were measured and the sling loads were identified. Each trip was sampled by considering every third sling, i.e., the first, fourth, seventh, etc., as the sample. Each trip was tested in two ways: the length frequencies of fish in the total trip and in the sample were compared using a Kolmogorov goodness-of-fit test (Conover, 1971), and the mean lengths of fish in the total trip and in the sample were compared by computing 95% confidence intervals. The representativeness of eight grab samples was examined similarly using data from two research cruises (grab sample 1 and 2) in which all fish were measured, and six commercial trips in which grab samples were taken from several systematically chosen slings. At the same time, every fish in these slings was measured. The lengths of all fish in the chosen slings were considered as the total population; the usual grab sample was considered an independent random sample. Details of these examinations are given in Table 1. The goodness-of-fit test compares the random sample with the total population. The critical value is that value which must be exceeded by the difference between the two distributions for the test to be significant, and the test value is the largest observed difference between the two distributions. In addition to providing a test of differences between means, the width of the confidence interval provides information on the variability of the sample.

Only one sling sample was significantly different from the total, and both the goodness-of-fit test and the comparison of means were significant. The relative variability in this sample was the highest of any of the samples, and the sample no doubt was too small to adequately represent the total. All other sling samples were nonsignificant, indicating no meaningful difference between the sample and the total catch. In the grab samples, none of the differences from commercial trips were significant, except for the failure of the confidence interval to contain the total mean in grab sample 5. However, both grab samples from the research cruises were significantly different. The samples of lengths from the research cruises represented a slightly smaller proportion of the total than the samples from the commercial trips and the smaller sample size may have contributed to the rejection of these grab samples. The grab sampling of the commercial trips was strictly supervised, and the people doing the sampling knew the technique was to be critically examined. The grab samples of fish in the research cruises were not taken for comparative purposes. Knowledge that a grab sample was to be critically examined would minimize the bias in a grab sample, and this statistical testing may not reflect the true situation. The samples from the landings of the research cruises may be more typical.

In theory, the sling sample reduces the bias due to improper selection of fish and demonstrates the acceptability of the sling sampling technique. Even though the testing did not show pronounced differences between the techniques, the potential for bias in the grab method is high, and sling sampling has been adopted as the standard within-vessel sampling technique. Beginning in 1973, sling sampling was used at each plant where it was feasible. At plants where problems were encountered, heading table constructions were modified or additional side tables were built.

Table 1. Results of testing sling and grab samples.

Sample	Number in Sample		Goodness-of-Fit Test		Mean Length (cm)		95% Confidence Interval
	Total	Sample	Critical Value	Sample Value	Total	Sample	
Sling							
1	621	212	.093	.035	125.9	126.6	123.7-129.5
2	507	176	.103	.036	135.7	136.7	133.6-139.8
3	507	175	.103	.033	127.4	128.8	125.6-132.0
4	547	196	.097	.046	122.9	123.6	120.5-126.7
5	441	155	.109	.089	128.0	130.9	126.6-135.2
6	142	61	.174	.061	127.8	129.1	123.9-134.3
7	470	164	.106	.118*	125.6	119.0	115.0-123.0*
8	862	201	.096	.044	94.4	94.8	83.4- 96.2
Grab							
1	2,489	579	.057	.180*	96.0	103.6	101.6-105.6*
2	2,529	478	.062	.090*	93.8	99.4	97.4-101.4*
3	256	62	.173	.147	87.9	94.7	87.7-101.7
4	681	141	.115	.041	92.4	91.1	87.8- 94.4
5	338	104	.133	.127	106.2	112.2	106.8-117.6*
6	655	169	.105	.057	80.6	81.8	79.7- 83.9
7	392	89	.144	.040	114.6	113.8	108.5-119.1
8	427	122	.123	.057	96.7	98.2	94.1-102.3

*Significant at the 5% probability level.

DOUBLE SAMPLING FOR AGE COMPOSITION

Double sampling for stratification is used extensively if one characteristic is relatively easy and inexpensive to measure and a second characteristic is more difficult and expensive to obtain (Cochran, 1963). Double sampling is often used to estimate length and age distributions in commercial fishery catches. Tomlinson (1971) discussed several methods of double sampling anchovy catches. Mackett (1963) compared double sampling with simple random sampling for determining age composition of Pacific albacore and concluded that double sampling was more efficient than simple random sampling for a given unit of cost of ageing and measuring fish. Kutkuhn (1963) also compared simple random sampling and double sampling with respect to age composition data for California salmon catches and concluded that direct random sampling is preferable to double sampling unless the cost of age samples is at least five times greater than the cost of length samples.

In double sampling halibut catches for length and age, the first sample, that for length, is assumed to be random with respect to the lengths in the catch. The subsample for age may be taken in several ways. Ketchen (1950) discusses two schemes, one (proposed by Fridriksson, 1934) in which the age sample is proportional to the length sample and another in which a constant number of otoliths is chosen at each length interval. In the latter scheme, all otoliths are taken at the tails of the length frequency distribution if the number present is less than the specified constant number. In past analyses, the relative age distribution of halibut in commercial landings was determined for each randomly chosen landing. A sample of lengths was taken from each landing and from this sample a second of fixed size was drawn for age determination; the several age frequencies were combined to represent either certain grounds or sections of the coast. This procedure is described below. An alternate approach also is described in which two changes in sampling procedure have been made. In this case, lengths originating in a common time-area block are combined before drawing the second sample, resulting in a single age sample which is proportional to the length sample. The alternate approach, which depends on between-vessel variability being less than within-vessel variability, was adopted in 1974 as a standard sampling procedure; in the following section, I show with empirical data that the between-vessel variability is much less than the within-vessel variability. To document analytical procedures used by IPHC for many years, I give details of the past as well as present techniques of sampling the catch for age data.

Past Method for Estimating Age Composition

Since the 1930's, sampling procedures used by IPHC to obtain age frequency data were largely intuitive rather than being developed systematically from a probability sampling design. IPHC assumed that the landing of a single vessel was a primary sampling unit, that the within-vessel variability in fish lengths was less than between-vessel variability, and that variability in either lengths or ages was large; hence, an age sample had to be taken from each length sample. Further, IPHC assumed that the vessels to be sampled were randomly chosen and that the length frequency distribution obtained from a landing was a random sample of the length frequency distribution of the vessel's catch. A subsample of the length frequency distribution from each landing was taken for age determination; here again, randomness of sampling was assumed. The assumption of large between-vessel variability was never tested, nor was the randomness of vessel choice. However, the assumptions of representative sampling for length frequencies within a landing and randomness in choosing the age subsample were examined from time to time. A large number of fish was measured from each landing to guard against non-randomness in sampling.

Age frequencies from vessels fishing the same grounds or subsections of the regulatory areas were combined by adding the individual age frequencies. A percentage age distribution was then obtained from the combined frequencies. The number of age k fish in the total catch was estimated by multiplying the "average" percentage of age k fish by the number of fish in the total catch. The latter was estimated from the overall average weight of the combined samples and the total catch. The concern for the primary sampling unit aspect of individual landings originally expressed in IPHC's sampling for age frequency data was ignored in the combining procedure. In fact, the landings with the greatest number of lengths carried the most weight in determining the percentage age distribution. Also, the use of number of fish in the catch rather than number of vessels in the fleet contradicted the concern for between-vessel variability. In any event, the variance for the estimated number of age k fish in the catch, or for the percentage of age k fish, was never computed.

Returning to the assumption that a vessel is a primary sampling unit, the assumption made by IPHC in past sampling, the following section gives an estimate of age k fish in the commercial catch and the variance of the estimate assuming a clustering effect attributable to the vessel's catch. Also, the assumption that the between-vessel variability in age was large relative to the within-vessel variability is examined with July 1973 data from Hecate Strait in Area 2 and the Kodiak region in Area 3. The annual catch from these regions represented 43% and 54% respectively of the total 1973 catch. This development follows directly from techniques for multistage cluster sampling and double sampling for stratification (Cochran, 1963; Raj, 1968). The total variability of the estimated number of age k fish is partitioned into between-vessel and within-vessel terms. The within-vessel term is further partitioned to estimate the variability of between-length and within-length strata. Let i be the subscript designating vessels (landings), j the subscript for length interval, and k that for age; and

B = number of boats fishing in subsection

b = number of boats in length frequency sample

N_i = number of fish in the catch of vessel i

$M_{i.}$ = number of fish in length frequency sample from vessel i

$m_{i.}$ = number of fish in age subsample of $M_{i.}$

The assumption of random sampling implies that the length frequency distribution of the i^{th} sample (landing) reflects the length frequency distribution of landings of commercial halibut. Schematically, data from vessel i would appear as in Table 2.

Table 2. Schematic presentation of length and age data.

Length	Age 1	Age 2	. . . . . .	Age k	Age Sample	Length Sample
l_1	M_{i11} m_{i11}				$m_{i1.}$	$M_{i1.}$
l_2	M_{i21} m_{i21}				$m_{i2.}$	$M_{i2.}$
. . .						
l_j				M_{ijk} m_{ijk}	$m_{ij.}$	$M_{ij.}$
Totals	$M_{i.1}$			$M_{i.k}$	$m_{i..}$	$M_{i..}$

An estimate of the number of age k fish in the catch of vessel i is given by

$$N_{i.k} = N_i \sum_j \left(\frac{M_{ij.}}{M_{i..}}\right)\left(\frac{m_{ijk}}{m_{ij.}}\right) \tag{1}$$

and the number of age k fish in the total catch by

$$A_k = \frac{B}{b} \sum_i N_{i.k} \tag{2}$$

Following the development of Raj (1968), an estimate of the variance of A_k is given by

$$V(A_k) = B^2 \left(\frac{1}{b} - \frac{1}{B}\right) Sb^2 + \frac{B}{b} \sum_i M_{i.}^2 \left(\frac{1}{m_{i.}} - \frac{1}{M_{i.}}\right) Sw_i^2 \tag{3}$$

where $\left(\frac{1}{b} - \frac{1}{B}\right)$ and $\left(\frac{1}{m_{i.}} - \frac{1}{M_{i.}}\right)$ are the finite population corrections,

$$Sb^2 = \frac{1}{b-1} \sum_i \left\{ N_{i.k} - \left[\frac{1}{b} \sum_i \left(N_{i.k} \right) \right] \right\}^2 \tag{4}$$

and Sw_i^2 is the within-vessel variance.

Consider now the within-vessel variance. Double sampling for stratification was used in sampling each vessel. The first sample determined the relative frequency of the length distribution and the second (for age) was taken from the first. The joint probability

$$p_{k(st)} = \sum_j \left(\frac{M_{ij.}}{M_{i..}} \right) \left(\frac{m_{ijk}}{m_{ij.}} \right) \tag{5}$$

gives the proportion of age k fish in the catch of the ith vessel. The within-vessel variance, i.e., the variance of $N_{i.k}$, is given by,

$$Sw_i^2 = V(N_{i.k}) = N_i^2 \, V_i \, (p_{k(st)})$$

where, following Cochran (1963),

$$V_i(p_{k(st)}) = \sum_j \left\{ \frac{p_{ij}^2 \, p_{ijk} \, q_{ijk}}{m_{ij} - 1} + \frac{p_{ij} \, (p_{ijk} - p_{ik(st)})^2}{M_{i..}} \right\} \tag{6}$$

$$\text{and } p_{ij} = \frac{M_{ij.}}{M_{i..}} \text{ and } p_{ijk} = \frac{m_{ijk}}{m_{ij.}} .$$

Interval estimates of A_k are obtained in the usual manner, i.e., $A_k \pm t_\alpha \sqrt{V(A_k)}$

where t_a is the critical value of the Student's distribution corresponding to a confidence coefficient of a.

The number of boats fishing, B, in a subsection of a regulatory area during any time interval was not known and had to be estimated. A simple proportion between the commercial fleet and the portion of the fleet sampled was used to estimate B. The variability introduced by this estimation was not included in the $V(A_k)$. The average age of fish in the ith trip and the variance of the average were estimated by standard weighting procedures.

Estimates of the number of age k fish and the variance of the estimates were computed for ages 6 through 12 and 8 through 14 for July 1973 data from Hecate Strait and the Kodiak region, respectively (Table 3; Appendix Tables 1 and 2). The variances were partitioned into between-vessel and within-vessel components. The within-vessel component was considerably larger than the between-vessel component. These data show that the assumption of large between-vessel variability was not valid and that additional variability was contributed to the estimated number of age k fish in the catch by using a sampling design based on that assumption.

The estimated variances in Table 3 are so large that little confidence can be placed in the estimated numbers of age k fish. The possibility that the within-vessel component was overestimated or is biased seems likely. As an alternative to the binomial assumption implicit in (1), within-vessel variances were computed using

$$Sw_i^2 = \frac{\sum_j (y_{ij} - \bar{y}_i)^2/(m_{i.k} - 1)}{M_{i.k} - 1} \tag{7}$$

where y_{ij} is the count of age k fish in the jth length interval. This variance was larger than that based on the binomial assumption. A reversal in the declining number of age k and age k + 1 fish in a cohort might be expected because of the large within-vessel variability. These reversals were evident in recent analyses, where total mortality was estimated for each succeeding year of cohort groups for Hecate Strait catches between 1957 and 1970. IPHC has assumed natural mortality to be approximately 0.2; in 4 of the 13 years between 1957 and 1970 the average total mortality for ages 11 to 15 was less than 0.2, in two additional years it was only slightly above 0.2. Such low estimates would be likely if the

Table 3. Estimated number, standard deviation of the estimate, and variance components of age k fish in the catch, July 1973.

Age	A_k	Standard Deviation	Variance Components Between-Vessel	 Within-Vessel
			Hecate Strait	
6	3,591	8,300	7,317	28,630
7	7,812	11,094	24,273	50,913
8	10,137	11,992	22,389	59,321
9	9,108	11,877	15,180	58,383
10	7,164	10,386	10,331	43,546
11	4,362	7,928	4,381	25,553
12	5,343	8,359	6,205	28,107
			Kodiak	
8	4,612	8,658	4,079	38,200
9	8,409	12,546	7,981	82,214
10	9,386	13,536	11,542	102,557
11	9,081	13,439	18,089	991,177
12	11,283	13,846	13,539	106,541
13	5,346	9,538	9,642	45,520
14	3,803	7,733	2,844	32,223

variability of the estimated numbers of age k fish in the catch were as large as indicated by these data.

The sampling for age frequencies was based on a fixed-size age sample (Ketchen, 1950), rather than a proportionate sample of the first length frequency sample. A fixed-size sample is unbiased and will be discussed in greater detail in the next section. To see that a fixed-size second sample does not introduce a bias in the estimated number at a specified age, assume as before that the population was sampled so that the length frequency distribution of the ith sample (landing) was proportional to the length frequency distribution of the population. The expected number of age k fish in the subsample is given by

$$E(M_{.k}) = \sum_j \left(M_{j.}\right) \left(\frac{m_{jk}}{m_{j.}}\right) \tag{8}$$

But $m_{jk}/m_{j.}$ is a random variable whose expected value is

$$E\left(\frac{m_{jk}}{m_{j.}}\right) = E(m_{jk}) \frac{1}{m_{j.}} = \frac{\overline{m}_{jk}}{m_{j.}}$$

because $m_{j.}$ is constant. On the average, $m_{jk}/m_{j.}$ is an unbiased estimate of the proportion of age k fish in length interval j. Looking at the problem from a large-sample viewpoint, as the age sample approaches the length sample in size, $m_{jk} \rightarrow M_{jk}$.

Because of the assumption of random sampling, $M_{jk}/M_{j.} \rightarrow \theta_{jk}/\theta_{j.}$, the population proportion, and

$$E(M_{.k}) = \sum_j \left(M_{j.}\right) \left(\frac{M_{jk}}{M_{j.}}\right) = \sum_j \left(M_{j.}\right) \left(\frac{\theta_{jk}}{\theta_{j.}}\right) = \theta_{.k}$$

as $M_{j.}$ becomes large.

Consequently, a fixed-size sample for age must conform to two criteria:

(1) The first sample must represent the population length frequency distribution, i.e.,

$$\frac{n_j}{n.} \doteq \frac{N_j}{N.}$$

(2) The second sample need not be proportional to the number in the length interval; however, the sample must be random with respect to age within each length stratum.

Present Method for Estimating Age Composition

Consider now the situation where the length frequency samples collected from a time-area block are combined before an age sample is drawn, the current method of sampling the landings for age. This is again double sampling for stratification, but only one age sample is drawn each month from the combined length frequencies; the number drawn is proportional to the number in the length sample. The otoliths selected for age determination are chosen randomly from the combined length frequencies in each length interval. Schematically the data would appear as in Table 2. Using notation similar to the previous section, the proportion of age k halibut in the catch is

$$p_{k(st)} = \sum_{j=1} p_j \, p_{jk} \qquad (9)$$

where j and k refer to length and age, respectively, (st) denotes a stratified estimate,

$$p_j = \frac{M_{j.}}{M_{..}} \quad \text{and} \quad p_{jk} = \frac{m_{jk}}{m_{j.}}$$

Note that the subscript i is omitted since the proportions are based on the combined length frequency sample. Because of the assumption of proportionality in the first stage sampling, p_j (an unbiased estimate of P_j, the population proportion of length j) can be regarded as a multinomial variate, and the variance of $p_{k(st)}$, assuming an infinite population, is estimated by

$$V(p_{k(st)}) = \sum_{j=1} \left(\frac{p_j^2 \, p_{jk} \, q_{jk}}{m_j - 1} + \frac{p_j \, (p_{jk} - p_{k(st)})^2}{M_{..}} \right) \qquad (10)$$

where the sample values of the proportions are used and $q_{jk} = (1 - p_{jk})$ (Cochran, 1963; Kutkuhn, 1963). The first term in the brackets represents the within-length strata variance; the second, between-length strata variance.

A point estimate of the number of age k halibut in the commercial catch is given by

$$N_{k(st)} = p_{k(st)} \, N^* \qquad (11)$$

where N* is the number of halibut in the total commercial catch. N* is unknown and must be estimated from the total landed weight and average fish weight from the sample of the catch, so

$$N^* = \frac{C_w}{\overline{w}} \qquad (12)$$

where C_W is the commercial catch in weight and

$$\overline{W} = \frac{\sum_i \sum_j \left[\overline{W}_j \; M_{ij.}\right]}{M_{...}}$$

$\overline{W}_j$ is the average weight of the jth length based on the length-weight regression (IPHC, 1972). IPHC has assumed that, on the average, the condition factor of halibut was constant and represented by a length-weight regression. An error is introduced with this assumption, but the practical problems of collecting and weighing halibut make annual recalculation of the length-weight regression unfeasible. Thus, an estimate of the variance of N * is given by

$$V(N^*) = C_W^2 \; V\left(\frac{1}{\overline{W}}\right) \tag{13}$$

The variability introduced by the length-weight regression is not considered here. An estimate of the variance of N_k to a first order approximation is given by

$$V(N_{k(st)}) = (p_{k(st)})^2 \; V(N^*) + (N^*)^2 \; V(p_{k(st)}) \tag{14}$$

An approximate interval estimate of N_k would be

$$N_{k(st)} \pm t_\alpha \sqrt{V(N_{k(st)})}$$

where t_a is the critical value corresponding to a confidence coefficient of *a*.

The effect of the two different sampling procedures — combining age frequencies from subsampling each length sample (Method I) or combining length frequencies and then subsampling for age (Method II) — is seen in the following calculation. Four samples were taken from the area-of-origin block 240-280 (Portlock block). The proportion of each age in the catch and the variance of these proportions for each sample are given in Table 4. Average proportions and the variance of these averages also are given. The variance reduction by Method II is pronounced at the younger ages and gains in importance when the size of the age sample is considered. Method II was based on 109 otoliths, Method I on 145 otoliths. A smaller variance was obtained with about a third fewer observations. The result indicates obvious benefits for the sampling and analysis of age data. Variances computed by the two methods using data obtained from sampling the landings from Hecate Strait in July 1973 and 1974 indicate that the reduction in variance might be greater than shown with these four samples.

The variance for Method II is the sum of two terms, one measures within-length strata variation and the other between strata variation. The first term is a function of the number of observations within the length strata, the second is a function of total length frequency sample size. Both of these are subject to control during sampling, and sample sizes can be chosen to minimize the variance of the proportion of age k fish in the time-area block.

Table 4. Proportion of each age in the catch ($\hat{p}$) and the variance ($V(\hat{p})$) for four samples, and the $\bar{p}$ and $V(\bar{p})$ obtained by different sampling procedures.

	Boat 1		Boat 2		Boat 3		Boat 4		Method I		Method II	
Age	$\hat{p}$	$V(\hat{p})$	$\hat{p}$	$V(\hat{p})$	$\hat{p}$	$V(\hat{p})$	$\hat{p}$	$V(\hat{p})$	$\bar{p}$	$\sqrt{V(\bar{p})}$	$\bar{p}$	$\sqrt{V(\bar{p})}$
6											.002	.0020
7			.114	.001688	.043	.000432			.058	.0228	.021	.0097
8	.122	.002019	.113	.001677	.054	.000955	.084	.000690	.084	.0333	.051	.0208
9	.196	.004291	.180	.003873	.181	.002550	.154	.002120	.177	.0530	.109	.0418
10	.214	.004516	.057	.00743	.249	.003169	.141	.002528	.191	.0584	.165	.0395
11	.064	.001489	.190	.00239	.074	.001067	.085	.001884	.087	.0377	.114	.0385
12	.180	.005242	.235	.004219	.160	.002109	.191	.003325	.181	.0557	.248	.0497
13	.084	.001610	.095	.001468	.077	.001128	.089	.001053	.084	.0348	.125	.0390
14	.057	.000711	.019	.000345	.083	.001181	.088	.000995	.071	.0318	.026	.0138
15	.055	.001078			.045	.000554	.077	.001206	.058	.0290	.038	.0163
16					.012	.000134	.043	.000742	.025	.0186	.033	.0151
17	.022	.000384			.022	.000364	.028	.000574	.024	.0207	.045	.0212
18											.015	.0118
19												
20							.021	.000141	.021	.0119		
21												
22											.003	.0026
Age:	m	35		20		50		40				
Length:	n	119		53		195		143				
	n^2	14,161		2,809		38,025		20,449				

ESTIMATION OF SAMPLE SIZE

In samples of commercial landings of halibut, lengths are obtained relatively easily and at much less expense than ages. Consequently, we need to know the number of ages that should be determined to estimate $p_{k(st)}$ with reasonable precision, that is, for a fixed-size length sample, M, what is the minimum size of the age sample, m. An estimate of sample size can be determined from the variance, $V(p_{k(st)})$, for each sampling procedure, i.e., a fixed-size age sample at each length interval, or an age sample that is proportional to the frequency of lengths in each length interval. Consider first the procedure used in the past by IPHC (the fixed-size age sample). In this procedure, all of the m_j's (the number of fish chosen from each length interval to be aged) were fixed, i.e.,

$$m_1 = m_2 = m_3 = \ldots m_j = m^*.$$

With this constraint, an estimate of M* is obtained directly from $V(p_{k(st)})$ by solving for m*:

$$m^* = \frac{M_{..} \, \Sigma \, p_j^2 \, p_{jk} \, q_{jk}}{M_{..} \, V - \Sigma \, p_j \, (p_{jk} - p_{k(st)})^2} + 1 \qquad (15)$$

V, the desired variance of $p_{k(st)}$, is obtained from the coefficient of variation, i.e.,

$$V(p_{k(st)}) = \left[(p_{k(st)}) \; (C.V.) \right]^2$$

where the coefficient of variation, C. V., is a specified percentage, thereby assuring that equal variability will be maintained for each estimate of sample size. Because it is the number of fish in each length interval chosen to be aged, m* must be multiplied by the number of length intervals to determine the total age sample size, m. . , for a time-area block. Obviously, this estimate would not apply in the tails of the length frequency distribution; it would be valid only for those length intervals where the number of lengths exceeded m_j^*. In the tails of the distribution, where the frequency was less than m*, all fish in each interval would be aged.

The degree of precision in any estimate depends on the size of the sample used to make the estimate. Data from Method II were used to determine m. . at 10% and 20% levels of precision, assuming a fixed-size age sample at each length interval. The age and length data from the sample are shown in Appendix Table 3, and m. . for two levels of precision are given in Table 5. Of course, there is no way of knowing with certainty if the variance of the $p_{k(st)}$'s based on samples of size m. . will, in fact, be small enough so that $p_{k(st)}$ will be within an acceptable range. However, m. . is derived from a confidence interval which is based on a specified probability of containing the parametric value. The term "level of precision" implies the use of a confidence interval in determining sample size. The maximum m. . estimated represents the overall sample size required to estimate all $p_{k(st)}$'s with a precision greater than some pre-chosen level. Meaningful estimates of m* for ages less than 8 or greater than 13 years were not determined due to the small size of $p_{k(st)}$ at these ages. At a 10% level of precision in estimating $p_{k(st)}$, approximately 2,000 otoliths (i.e., m. .) would be needed for each time-area block. When the level of precision was changed to 20%, m. . (the number of otoliths required) was reduced to approximately 450.

Consider next the procedure presently used by IPHC in which the number of fish to be aged in each length interval is proportional to the frequency in each length interval, i.e.,

$$\frac{m_{j.}}{m_{..}} \doteq \frac{M_{j.}}{M_{..}}$$

and $m_{j.} \doteq (m_{..})(p_{j.})$. Kutkuhn (1963) gives $m_{j.}$ obtained by solving $V(p_{k(st)})$ for $m_{j.}$.

Table 5. Estimates of sample size for different sampling designs and levels of precision based on data from four samples from Portlock block.

Age	±10%		±20%	
	Fixed	Proportional	Fixed	Proportional
9	1,780	808	402	174
10	975	438	240	100
11	2,042	758	462	165
12	595	267	162	66
13	1,622	664	395	154

Usually the m_j's would be chosen proportional to the within-length strata variance, i.e.,

$$m_j = \frac{M_{..}\, p_j\, Sw_j}{\sum\limits_j p_j\, Sw_j} \tag{16}$$

Optimum values of $m_{..}$ and $M_{..}$ can be obtained when these values are used in conjunction with the cost function,

$$C = m_{..}C_m + M_{..}C_M \tag{17}$$

where C is total cost and C_m and C_M are respective costs of obtaining age and length frequencies (Cochran, 1963). Costs of sampling for age and length frequencies have never been determined, and the rough estimates of these which are available are inadequate for use in estimating sample sizes. Even though the $m_{j.}$'s as developed by Kutkuhn may not give minimum or optimum variance, they do serve to compare the two sampling designs for choosing the age sample. Data from Method II (Appendix Table 3) were used again to estimate $m_{..}$. The $m_{..}$'s also are given in Table 5. The reduction in $m_{..}$ was large when the age sample was proportional to the length sample. The fixed-size age samples require from two to three times as many observations for the same level of precision as proportional age samples. At present, approximately 300 otoliths are aged per month from trips originating in the different area blocks. This corresponds to a level of precision in $p_{k(st)}$ somewhat in excess of 20% if fixed-size age sampling was used, but approximately 12-15% precision if proportional age sampling was used. IPHC has adopted the proportional age sample as the standard procedure because of this difference in precision in $p_{k(st)}$.

Number of Fish Per Landing

Occasionally, length frequency data from specified grounds within groupings of statistical divisions are used for special analyses. Length data from IPHC research cruises were used to study the representativeness of uniform sampling throughout the entire unloading and the effect of sampling only fractions of the vessel's catch. All fish caught during these cruises were measured, and the chronological order of capture was recorded. The research cruises were made with chartered commercial vessels using conventional setline gear. In the uniform sampling scheme, every l^{th} fish was included in the sample, in the fractional sampling scheme the catch was divided chronologically into l parts and all fish in the fraction were considered to be the sample. Chi-square analysis was used to test the hypothesis that the length frequency distribution of the sample was the same as the length frequency distribution in the total catch. Sample sizes and x^2 values for length data obtained from a single research cruise to Portlock grounds (statistical divisions 240-280) are given in Appendix Table 4a. Total trip size was 478 fish, every l^{th} fish ($l = 2, 3, 4, 5$) was chosen. In no case was the hypothesis rejected at the 5% level.

The total catch was next divided into halves and then thirds. Sample sizes and x^2 values are given in Appendix Table 4b. In the analysis based on halves, sample size was 239 and neither x^2 was significant. In the analysis based on thirds, sample size was 160 and x^2 for the first and second thirds was significant while that for the last third was not. These sample sizes and x^2 values are typical of several sets of data from research cruises. They imply that if a trip is sampled uniformly, samples as small as 100 fish will represent the trip. If a fraction of the trip is sampled, approximately 200 fish are required to represent the total trip. Therefore, inasmuch as sling sampling involves every fish in part of the load, a sling sample of 200 fish should be representative of a vessel's catch.

ESTIMATING AGE COMPOSITION FOR REGULATORY AREAS

Management decisions usually are based on data for entire regulatory areas rather than on portions of them; consequently, an average age frequency for a regulatory area is needed. The age frequencies from each time-area block are averaged, and the catch from each block is used as the weighting factor.

Let

W_{ht} = catch from area block h during month t within a specified regulatory area

$W_{..} = \sum_h \sum_t W_{ht}$ = catch from the regulatory area during the year

then

$$N_{k(r)} = \frac{\sum_{h=1}^{H} \sum_{t=1}^{T} W_{ht} N_{k(st)h}}{W_{..}} \qquad (18)$$

The variance of $N_{k(r)}$ is given by

$$V(N_{k(r)}) = \frac{\sum_{h=1}^{H} \sum_{t=1}^{T} W_{ht}^2 \,(V(N_{k(st)h})}{W_{..}^2} \qquad (19)$$

where $V(N_{k(st)h})$ is given by (14).

Combinations of age frequencies for regulatory areas for specified months could be obtained as above with the proper adjustment of the weighting factor W_{ht}.

DISCUSSION AND SUMMARY

A probability model for sampling commercial landings of halibut to determine age composition of the catch is presented. The model is stratified according to months and groups of statistical divisions. The entry of a vessel into port is considered a random event, and a random choice of vessels for sampling is based on a systematic selection of vessels as they unload. Random within-vessel samples for length frequencies are obtained by systematically selecting sling loads of halibut as they are unloaded; the slings are assumed random with respect to each other. A double sampling procedure is used; the first sample, that for length frequency, is random with respect to lengths. The second, that to be aged, is proportional to the length frequency sample. The age sample is selected from the combined length frequencies within a time-area block. Age and length frequencies for regulatory areas are expressed as weighted averages of the frequencies from the groups of statistical divisions. Estimates of sample size for stated levels of precision in the relative age frequency are given.

The improved random sampling design and method of analysis described in this report were implemented in 1974 and represent a departure from earlier approaches. The previous design is discussed, and estimates of number and variance of age k fish in the catch are given, assuming a clustering effect due to individual vessel catches. I demonstrate that the within-vessel variability was greater than the between-vessel variability; hence, use of a vessel as a primary sampling unit was dropped. The manner by which vessels in the current sampling design are chosen for sampling, as well as the manner by which sling loads of halibut are chosen, differ from the procedures used before 1974. Moreover, the method of drawing the age sample subsequent to the combination of data also differs; in the current procedure, length frequencies are combined for a time-area block and a single age sample is drawn, rather than drawing an age sample from each length sample and combining these. In the present method, the estimated variance of the relative age frequency is reduced.

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LITERATURE CITED

Bell, F. Heward, Henry A. Dunlop, and Norman L. Freeman

1952 Pacific Coast halibut landings 1888 to 1950 and catch according to area of origin. International Fisheries Commission, Report No. 17, 47 p.

Cochran, William G.

1963 Sampling techniques. John Wiley and Sons, Inc., Second Edition, New York, 413 p.

Conover, W. J.

1971 Practical nonparametric statistics. John Wiley and Sons, Inc., New York, 462 p.

Fridriksson, A.

1934 On the calculation of age-distribution within a stock of cod by means of relatively few age-determinations as a key to measurements on a large scale. Conseil Permanent International pour l'Exploration de la Mer, Rapports et Proces-Verbaux des Reunions, Volume 86, No. 6, pp. 1-14.

Hardman, William H., and G. Morris Southward

1965 Sampling the commercial catch and use of calculated lengths in stock composition studies of Pacific halibut. International Pacific Halibut Commission, Report No. 37, 32 p.

International Pacific Halibut Commission

1951 Regulation and investigation of the Pacific halibut fishery in 1949 (Annual Report). International Fisheries Commission, Report No. 15, 24 p.

1961 Regulation and investigation of the Pacific halibut fishery in 1960 (Annual Report). International Pacific Halibut Commission, Report No. 30, 24 p.

1972 Length-weight relationship. International Pacific Halibut Commission, Information Bulletin No. 3, 1 p.

1973 Annual Report 1972. International Pacific Halibut Commission, 36 p.

Ketchen, K. S.

1950 Stratified subsampling for determining age distributions. Transactions of the American Fisheries Society, Volume 79, pp. 205-212.

Kutkuhn, Joseph H.

1963 Estimating absolute age composition of California salmon landings. California Department of Fish and Game, Fish Bulletin No. 120, 47 p.

Mackett, David J.

1963 A method of sampling the Pacific albacore (*Thunnus germo*) catch for relative age composition. Proceedings of the World Scientific Meeting on the Biology of Tunas and Related Species, Experience Paper, Volume 20, pp. 1355-1366.

Myhre, Richard J.

1969 Gear selection and Pacific halibut. International Pacific Halibut Commission, Report No. 51, 35 p.

Raj, Des

1968 Sampling theory. McGraw-Hill Book Company, New York, 302 p.

Skud, Bernard Einar

1972 A reassessment of effort in the halibut fishery. International Pacific Halibut Commission, Scientific Report No. 54, 11 p.

Southward, G. Morris, and William H. Hardman

1973 Otolith length and fish length of Pacific halibut. International Pacific Halibut Commission, Technical Report No. 10, 10 p.

Thompson, William F., Harry A. Dunlop, and F. Heward Bell

1931 Biological statistics of the Pacific halibut fishery (1) changes in yield of a standardized unit of gear. International Fisheries Commission, Report No. 6, 108 p.

Tomlinson, Patrick K.

1971 Some sampling problems in fishery work. Biometrics, No. 27, pp. 631-641.

APPENDIX

Appendix Table 1. Hecate Strait, July 1973 data.

							P_k				Ages:		$S'w_i^2$				
Boat No.	$\hat{N}_i$	$m_{i..}$	$M_{i..}$	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	6	7	8	9	10	11	12
1	1,677	50	230	.0351	.0328	.1002	.2380	.1351	.0904	.0395	.00474	.00929	.00786	.00954	.00846	.00207	.00113
2	1,081	50	230	.1683	.1937	.3786	.1319	.0254	.0145	.0580	.00490	.00691	.01513	.00580	.00064	.00482	.00861
3	851	50	179	.2384	.1904	.2714	.1215	.0754	.0000	.0112	.01114	.00678	.01704	.00522	.01461	.00000	.00043
4	348	20	80	.0000	.2750	.1750	.1875	.0500	.1000	.0750	.00000	.03875	.02750	.02735	.01187	.02844	.01156
5	1,563	40	209	.1906	.3286	.1196	.1467	.0861	.0287	.0191	.00589	.01161	.00859	.00733	.00266	.00103	.00951
6	912	40	184	.0543	.0779	.0996	.1042	.0901	.0489	.0399	.00758	.00310	.00494	.00476	.00471	.00192	.00168
7	1,316	38	187	.0490	.1460	.2449	.2020	.0160	.0606	.0738	.01128	.00949	.02019	.00673	.00142	.00171	.00816
8	1,500	62	270	.0580	.1383	.1326	.1522	.1264	.1579	.1482	.00889	.00736	.00516	.00393	.00409	.01099	.00548
9	423	20	99	.0000	.3687	.3182	.0354	.1162	.0859	.0758	.00000	.02026	.02078	.00297	.03291	.02266	.02322
10	1,003	40	187	.1182	.0906	.2116	.1259	.2070	.1043	.1210	.00461	.00278	.00909	.00478	.01795	.00619	.02090
11	1,681	70	292	.0000	.0354	.0628	.0628	.1207	.0342	.1601	.00000	.00438	.00502	.00202	.00313	.00085	.00695
12	1,673	50	222	.0323	.0282	.3529	.2616	.1059	.0901	.0931	.00097	.00084	.01101	.01665	.00353	.00799	.01149
13	461	26	86	.0000	.1042	.0000	.1279	.2907	.0756	.1221	.00000	.01851	.00000	.00970	.03390	.00688	.01810
14	1,005	50	235	.0000	.0653	.1039	.1667	.2018	.1525	.1649	.00000	.00232	.00681	.00554	.07112	.00605	.00651
15	335	35	149	.0067	.0537	.0761	.2640	.1879	.1096	.0604	.00666	.01306	.00398	.01203	.01516	.00547	.00408
16	827	43	188	.0000	.0665	.1755	.2207	.1773	.1028	.1082	.00000	.00328	.00685	.00988	.01281	.01021	.01047
17	217	35	104	.0000	.1298	.2324	.0481	.1282	.0673	.1346	.00000	.02110	.02425	.00518	.00903	.00461	.02090
18	1,570	60	280	.0375	.0165	.2191	.2007	.2630	.1080	.1177	.00118	.00029	.00568	.00566	.00639	.00682	.00944

$$S'w_i^2 = \sum_j \left\{ \left(\frac{M_{ij.}}{M_{i..}}\right)^2 V\left(\frac{m_{ijk}}{m_{ij.}}\right) + \left(\frac{m_{ijk}}{m_{ij.}}\right)^2 V\left(\frac{M_{ij.}}{M_{i..}}\right) \right\}$$

$$B = \frac{(1276276)(18)}{425823} = 53.95 \rightarrow 54$$

Appendix Table 2. Kodiak, July 1973 data.

Boat No.	$\hat{N}_i$	$m_{i..}$	$M_{i..}$	P_k							$S'w_i^2$						
				P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	Ages: 8	9	10	11	12	13	14
1	1,228	40	172	.1017	.0620	.1366	.0581	.2112	.1376	.1027	.00890	.00247	.00694	.00243	.01114	.01620	.01411
2	896	30	103	.0631	.1311	.1165	.1392	.2638	.0291	.0809	.01268	.01699	.01839	.00940	.01791	.00942	.00577
3	1,226	50	213	.1588	.1064	.1455	.0595	.1882	.0485	.0250	.01177	.00721	.00616	.00296	.00673	.00189	.00212
4	1,613	50	269	.0421	.1078	.1691	.1044	.1806	.1131	.0703	.00149	.00378	.00593	.00310	.00621	.00374	.00327
5	1,204	50	200	.0596	.1971	.0996	.1350	.2450	.0642	.0567	.01135	.00616	.00382	.00423	.01230	.00670	.00229
6	1,323	51	205	.0902	.1435	.1439	.1374	.0878	.0634	.0691	.00463	.00489	.00584	.00528	.00864	.00214	.00745
7	296	16	30	.0000	.2500	.1000	.1830	.1000	.0000	.1000	.00000	.07538	.06332	.04432	.06332	.00000	.09657
8	2,706	45	224	.0480	.1603	.1570	.1615	.1577	.1421	.0513	.00232	.01016	.00478	.00565	.00564	.00613	.00679
9	2,474	45	210	.1048	.1095	.1206	.2016	.2064	.0000	.0698	.00654	.00972	.00885	.00799	.01922	.00000	.01126
10	1,027	40	162	.0370	.1358	.1420	.3169	.0988	.1049	.0268	.00742	.01279	.00606	.01878	.00938	.01068	.00111
11	1,237	16	45	.0000	.1111	.3000	.0222	.1556	.1111	.1333	.00000	.04196	.05993	.02171	.02927	.04196	.04146
12	808	35	141	.0603	.1938	.1797	.1324	.1785	.0803	.0331	.00424	.01415	.01096	.01159	.01650	.00447	.00144
13	1,110	38	162	.1286	.1564	.1337	.2376	.1862	.0648	.0062	.01843	.01187	.00718	.01651	.01668	.00971	.00616
14	1,103	30	158	.0918	.1266	.1171	.0601	.1899	.2532	.0538	.01053	.00702	.00802	.00357	.00919	.02020	.00973
15	1,169	30	114	.0614	.1316	.1579	.2018	.2325	.0219	.0000	.00508	.01859	.01035	.03021	.02212	.00234	.00000
16	1,314	45	216	.0386	.1559	.2022	.1867	.0957	.0910	.0849	.00527	.00750	.00725	.00740	.00345	.00731	.00819
17	688	40	143	.0839	.1538	.1410	.0851	.1911	.0886	.0874	.00757	.01939	.00666	.00329	.00924	.01124	.01211
18	527	35	151	.1071	.0497	.0662	.0673	.1910	.0199	.0960	.00602	.00272	.00420	.00359	.01114	.00650	.00530
19	564	35	150	.1067	.1289	.1000	.2111	.2933	.1000	.0200	.00612	.00674	.01237	.01217	.02164	.01287	.01322

$$S'w_i^2 = \sum_j \left\{ \left(\frac{M_{ij.}}{M_{i..}}\right)^2 V\left(\frac{m_{ijk}}{m_{ij.}}\right) + \left(\frac{m_{ijk}}{m_{ij.}}\right)^2 V\left(\frac{M_{ij.}}{M_{i..}}\right) \right\}$$

$$B = \frac{(2604257)(19)}{935608} = 52.89 \rightarrow 53$$

Appendix Table 3. Distribution of length and age data for four Portlock block samples.

Length Interval (mid-point)	n_j	Age														m_j
		6	7	8	9	10	11	12	13	14	15	16	17	18	22	
77	3		1	1		1										3
82	2	1	1													2
87	9		1		2	1	1	1								6
92	22		2	3	1											6
97	29			1	3	2										6
102	35				1	3		2								6
107	53			1		2	1	2								6
112	65				1		2	2	1							6
117	56				1	1	1		2				1			6
122	47					2		1	2		1					6
127	47				1		2	2	1							6
132	39					1		2	1				1	1		6
137	34							3		1		1	1			6
142	27					1		2		1		2				6
147	17							3	1		1					5
152	9					1			1	1	3					6
157	8									1	2		1	1	1	6
162	3						1	2								3
167	2							1								1
172	2											1				1
177	–															–
182	–															–
187	1										1					1
	510															100
$p_{k(st)}$		.002	.021	.051	.109	.165	.114	.248	.125	.026	.038	.033	.045	.015	.003	

Appendix Table 4a. **Overall comparison of body length frequencies by x^2 test for the first type of subsample (every lth measurement), data from Portlock block.**

N = 478 (d.f. = 17)

Subsample	$l = 2$	$l = 3$	$l = 4$	$l = 5$
n	239	159	119	96
x^2	8.1356	15.1958	13.3225	16.8741
Probability	$.98 > P > .95$	$.70 > P > .50$	$.80 > P > .70$	$.50 > P > .30$

Appendix Table 4b. **Overall comparison of body length frequencies by x^2 test for second type of subsample, (l divisions), data from Portlock block.**

N = 478 (d.f. = 17)

l	Subsamples	n	x^2	Probability
$l = 2$	1st half	239	8.0047	$.98 > P > .95$
	2nd half	239	7.8871	$.98 > P > .95$
$l = 3$	1st third	159	28.7708	$.05 > P > .02$*
	2nd third	159	27.5060	$.10 > P > .05$*
	3rd third	160	10.0381	$.90 > P > .80$